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VISUALIZATION OF TRENDS AND FLUCTUATIONS IN CLIMATIC RECORDS[1]

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ABSTRACT: Small systematic changes in climatic records are often poorly visualized by standard time series plots because they are usually hidden by the magnitude and variability of the data values themselves. A visualization approach based on the rescaled adjusted partial sums (RAPS) which overcomes the above-stated shortcomings is presented. This visualization highlights trends, shifts, data clustering, irregular fluctuations, and periodicities in the record. Additional information on the number, magnitude, shape, frequency, and timing of fluctuations and trends can also be inferred. The visualization approach can be used for preliminary visual inspection of a time series, to gain a feel for the data, and/or to guide and focus subsequent statistical tests and analyses. It is not intended as a substitute for standard statistical analysis. Alternatively, the visualization approach can be used to display findings of a time series analysis. The capabilities and limitations of the approach are discussed and illustrated for two time series of annual rainfall values.
(KEY TERMS: annual precipitation; rescaled adjusted partial sums.)

INTRODUCTION

A time series analysis can detect and quantify trends and fluctuations in climatic records (Yevjevich, 1972; Chatfield, 1984; Salas *et al.*, 1980). The commonly used methods for analyzing trends and cyclical variations in a time series are regression and spectral analysis, respectively. However, it is not always clear at the onset of the analysis what trends and fluctuations to look for, when they occur in time, how large they are with respect to the background variability of the data, and which test to apply to detect them (Klemes, 1978). A preliminary visual inspection of a time series can provide valuable information about the structure of the series so that a focused analysis can be conducted. However, a simple time plot of a climatic variable does not always lead to good visualization, particularly when the object is the determination of subtle changes over time. Such changes are often masked by the magnitude and variability of the data values themselves (Chatfield, 1984).

The concealing effect of the mean and variability of the data on the visualization of comparatively small changes is eliminated by rescaling the variable and summing the resulting departures about the mean over time. Positive and negative departures largely cancel each other during the summation, leaving cumulative differences that are primarily related to the trends and fluctuations of the record. These cumulative differences lend themselves to visualization because the summation of consistent small departures spanning several time increments leads to large cumulative departures which can easily be recognized. This type of summation is often referred to as cumulative departures or rescaled adjusted partial sums (RAPS). The approach has been used in conjunction with the design of storage reservoirs (Rippl, 1883; Hurst, 1951; Jackson, 1975; Chow, 1964); earthwork planning (Sain, 1976); testing for homogeneity and shifts in time series (Hawkins, 1977; Worsley, 1979; Buishand, 1982, 1984); evaluation of the range of a time series (Salas-La Cruz and Boes, 1974); and identification of cyclical patterns (Williams, 1961).

The RAPS are not without shortcomings. They have been shown to be the first step in cumulative processes that ultimately lead to apparent periodicities that in reality do not necessarily exist (Feller, 1966; Klemes and Klemes, 1988; Yule, 1926). Klemes and Klemes (1988) caution against using partial sums for the display of empirical records because of the rapid convergence of higher order partial sums to the

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sinusoidal limit and the resulting impression that the record is governed by a sinusoidal periodicity. This tendency is particularly true when summing certain natural processes, such as fluctuations in glacier volume or water levels in large lakes because they are themselves already the result of an accumulation process. Applying the RAPS to these processes results in higher order cumulative processes (Klemes and Klemes, 1988).

On the other hand, many fluctuations in a time series are real and lend themselves to visualization by use of the RAPS. Even though the RAPS approach has been used in the past, it is not widely applied to visualize and identify patterns in time series. This paper aims at reintroducing the approach as a practical tool for visualization and recognition of apparent trends and fluctuations in a time series of climatic records. Visualization often provides important clues as to the nature of the patterns and their location in the time series. This in turn can direct an investigator to conduct specific tests on pertinent portions of the record. The visualized pattern identification is to be considered as a complement to standard statistical tests. The fundamentals of the RAPS approach are presented first, followed by several hypothetical examples and by two applications involving annual rainfall records. Finally, the capabilities and limitations of this visualization approach of climatic records are discussed. In the following discussion, periodic and irregular oscillatory variations over several time increments are simply referred to as fluctuations, and unidirectional variations over longer time periods are referred to as trends.

FUNDAMENTALS

Let $Y = \{Y_t; t = 1, \ldots, n\}$ represent a time series of a climatic variable with expected value μ_t and variance σ^2, where μ_t may be a function of time and σ^2 is assumed constant. Both μ_t and σ^2 are generally unknown for climatic records. It is further assumed that $\{Y_t\}$ is normally distributed, or can be transformed into a normal distribution to avoid biases that may result from asymmetric distributions. The selection of an appropriate transformation prior to analysis is discussed in Legates (1991). The RAPS of the $\{Y_t\}$ are defined as follows:

$$X_k = \sum_{t=1}^{k} \frac{Y_t - \overline{Y}}{S_Y}; \; k = 1, \ldots, n \tag{1}$$

where $\overline{Y}$ is sample mean; S_Y^2 is variance; n is number of values in the time series; and k is counter limit of the current summation. The plot of the RAPS versus time is the visualization of the trends and fluctuations of $\{Y_t\}$. In the following, properties of the RAPS to meet the visualization objectives are discussed for several hypothetical and real-life applications.

CASE 1

In Case 1, $\{Y_t\}$ is assumed to be independently and identically distributed (IID), thus having no trends or fluctuations in its expected value. Given these conditions, the characteristics of the RAPS are as follows (Salas-La Cruz and Boes, 1974):

$$E[X_k] = 0 \tag{2}$$

$$VAR[X_k] = VAR\left[\frac{1}{\sigma}\left(\sum_{t=1}^{k} Y_t - \frac{k}{n}\sum_{t=1}^{n} Y_t\right)\right]$$

$$= \frac{k\,(n-k)}{n} \tag{3}$$

Equations (2) and (3) show that an IID time series results in RAPS having an expected value of zero (no trends or fluctuations) and in a parabolic relation for its variance with the highest value when k = n/2 and lowest when k = n.

A time series $\{Y_t\}$ of 100 values generated randomly from a normal distribution with mean 40 and variance 100 and its corresponding RAPS are shown in Figure 1. Decreasing patterns in the RAPS are the result of periods of mostly below-average Y_t values, whereas increasing patterns are the result of periods of mostly above-average Y_t values. The sustained departure in the RAPS at the beginning of the plot is the result of a period of average Y_t values preceded by consecutive above-average and terminated by consecutive below-average Y_t values. It is important to recognize that the time period of the sustained departure corresponds to a period of average Y_t values. Therefore, sustained departures in the RAPS are not very relevant to the determination of change; the increasing/decreasing patterns, corresponding slopes and time are the most informative, because they describe sustained changes in the Y_t values. These changes are not readily apparent in the standard time series plot, but are clearly identified in the RAPS.

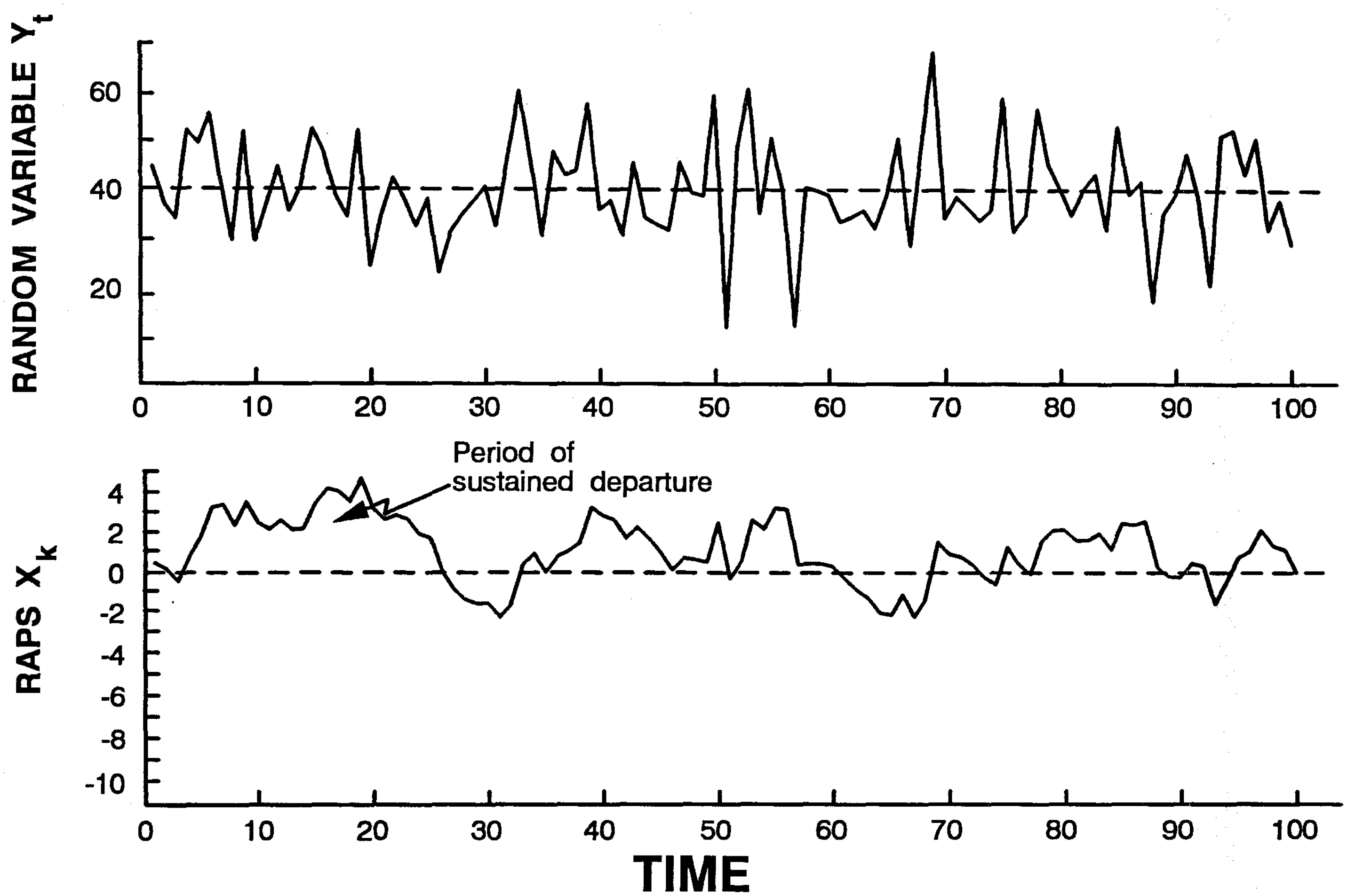

Figure 1. Time Series of n = 100 Values of Randomly Generated Y_t Values (top), and Corresponding Partial Sums X_k (bottom).

CASE 2

In Case 2, the previous time series is amended by including a sudden shift δ in the expected value at time m during the period of record. Such a shift in a climatic record may be the result of station relocation, change in instrumentation or station exposure, a sudden alteration in micrometeorologic conditions due to nearby human activity, or a real climatic change. The expected value of the corresponding RAPS is as follows:

(a) for t = 1, . . . m and k ≤ m

$$E[X_k] = \frac{1}{\sigma}\left[\sum_{t=1}^{k} E[Y_t] - \frac{k}{n}\sum_{t=1}^{n} E[Y_t]\right]$$

$$= -\frac{\delta}{\sigma}\,\frac{k(n-m)}{n} \tag{4}$$

(b) for t = m+1, . . . , n and k ≥ m

$$E[X_k] = \frac{1}{\sigma}$$

$$\left[\sum_{t=1}^{m} E[Y_t] - \frac{k}{n}\sum_{t=1}^{m} E[Y_t] - \frac{k}{n}\sum_{t=m+1}^{n} E[Y_t]\right]$$

$$= -\frac{\delta}{\sigma}\,\frac{m(n-k)}{n} \tag{5}$$

The variance VAR[X_k] remains unaffected by the shift and is still given by Equation (3). Equations (4) and (5) show that the expected value of the RAPS is a linear function of time k given the magnitude and timing of the shift. Therefore, a shift in a climatic record is identified by the existence of a linear trend in the RAPS which reverses direction at the time at which the shift occurs.

As an illustration, the example given in Case 1 is amended by adding a shift of 10 percent of the original mean to all Y_t values after time 60. The shift is not visually apparent in the standard time series plot (top of Figure 2). However, a shift is clearly defined in the plot of the RAPS (bottom of Figure 2) by a consistent downward trend followed by a consistent upward trend. This visual trend does not prove the existence of a shift, but does draw attention to a feature that requires further analysis such as, for example, a homogeneity test or a test of hypothesis for the shift. It is important to recognize that if the shift occurs within the body of the record, as in this example, its identification is usually straightforward. However, if the shift occurs within a few time steps of either end of the record, the distinction between a shift and a short-lived variation becomes virtually impossible.

CASE 3

In Case 3, the basic assumptions of Case 1 are amended to include a continuous uni-directional linear trend. Such a trend may reflect the effect of urbanization, a gradually changing environment, or a gradual shift in instrument recording accuracy. The expected value of the RAPS is as follows:

$$E[X_k] = \frac{1}{\sigma}\left[\sum_{t=1}^{k} E[Y_t] - \frac{k}{n}\sum_{t=1}^{n} E[Y_t]\right]$$

$$= \frac{1}{\sigma}\left[k\mu_o + \delta\sum_{t=1}^{k} t - \frac{k}{n}\left(n\mu_o + \delta\sum_{t=1}^{n} t\right)\right]$$

$$= -\frac{\delta}{\sigma}\left[\frac{k(k+1)}{2} - \frac{k}{n}\frac{n(n+1)}{2}\right] = \frac{\delta}{2\sigma}\left(k^2 - kn\right) \qquad (6)$$

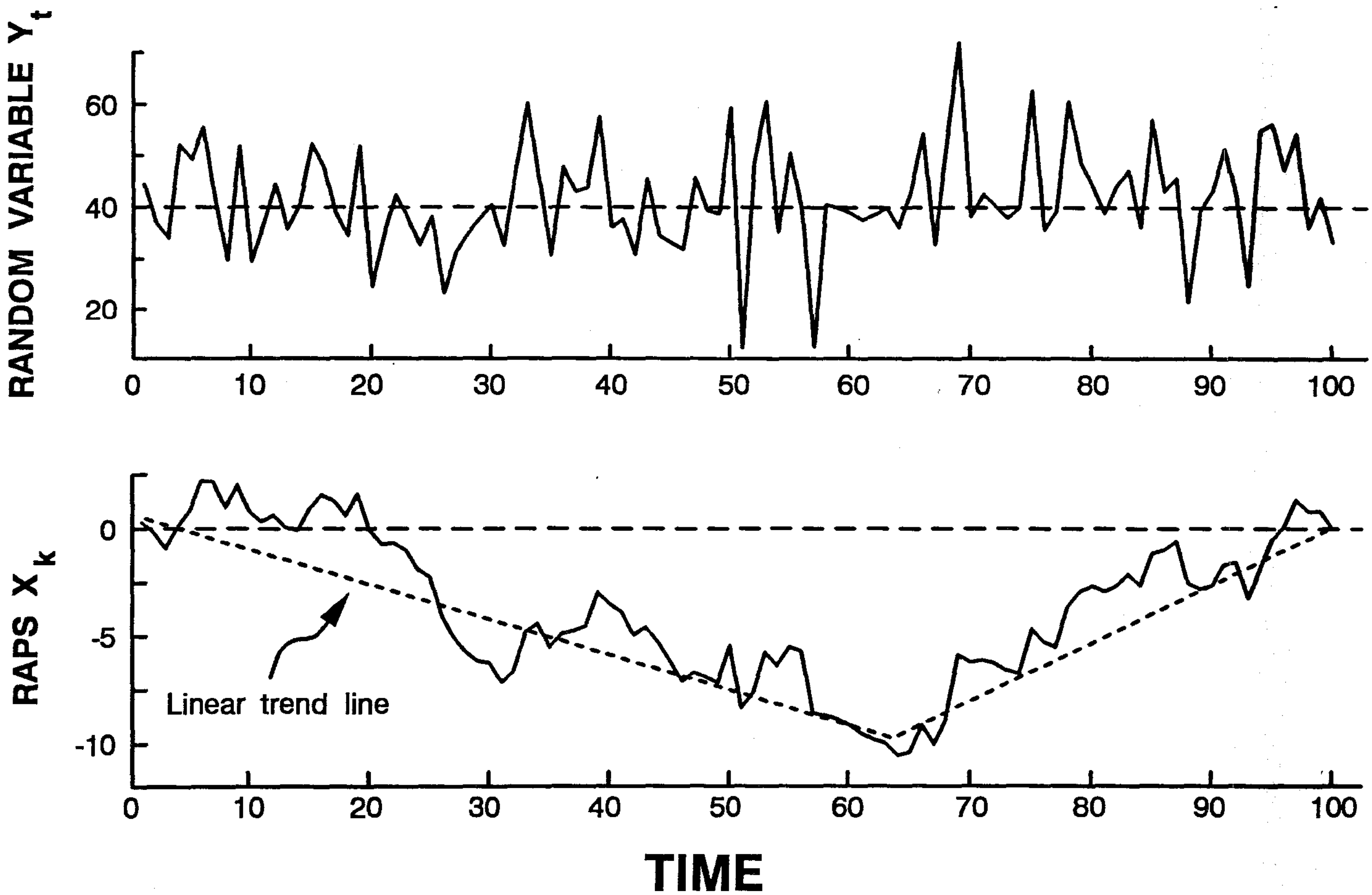

Figure 2. Time Series of n = 100 Values of Randomly Generated Y_t Values With Shift at Time t=60 (top), and Corresponding Partial Sums X_k (bottom).

Where μ_o is the expected mean of $\{Y_t\}$ at time 0 and δ is the trend constant; the expected mean of $\{Y_t\}$ at any time is μ_o plus (d*t). The variance VAR[X_k] is unaffected by the uni-directional trend and is given by Equation (3). This equation shows that a uni-directional trend in the time series leads to a parabolic trend in E[X_k] with the RAPS varying around this trend.

As an example, the Y_t values in Case 1 are modified by adding 0.05*t. A visual inspection of the standard time series also suggests that the values at small times are somewhat lower than those at large times, but the trend is far from obvious (top of Figure 3). In contrast, the plot of the RAPS shows a downward parabolic trend suggesting the existence of a possible trend which could be tested for using, for example, standard regression tests or a hypothesis test for a trend.

GENERAL CASE

In a more general case, $\{Y_t\}$ is nonstationary with respect to μ_t. The variance VAR[Y_t] = σ^2 is again assumed constant. With these initial conditions, the expected value for the RAPS is given as follows:

$$E[X_k] = \frac{1}{\sigma}\left[\sum_{t=1}^{k} E[Y_t] - \sum_{t=1}^{k} E[\bar{Y}]\right]$$

$$= \frac{1}{\sigma}\left[\sum_{t=1}^{k} \mu_t - \frac{k}{n}\sum_{t=1}^{n} \mu_t\right] \tag{7}$$

The corresponding variance VAR[X_k] is still given by Equation (3). Equation (7) is a generalization of the previous cases. It indicates that the expected value of the RAPS depends on the form of the mean function, μ_t. By specifying the form of the mean function, μ_t,

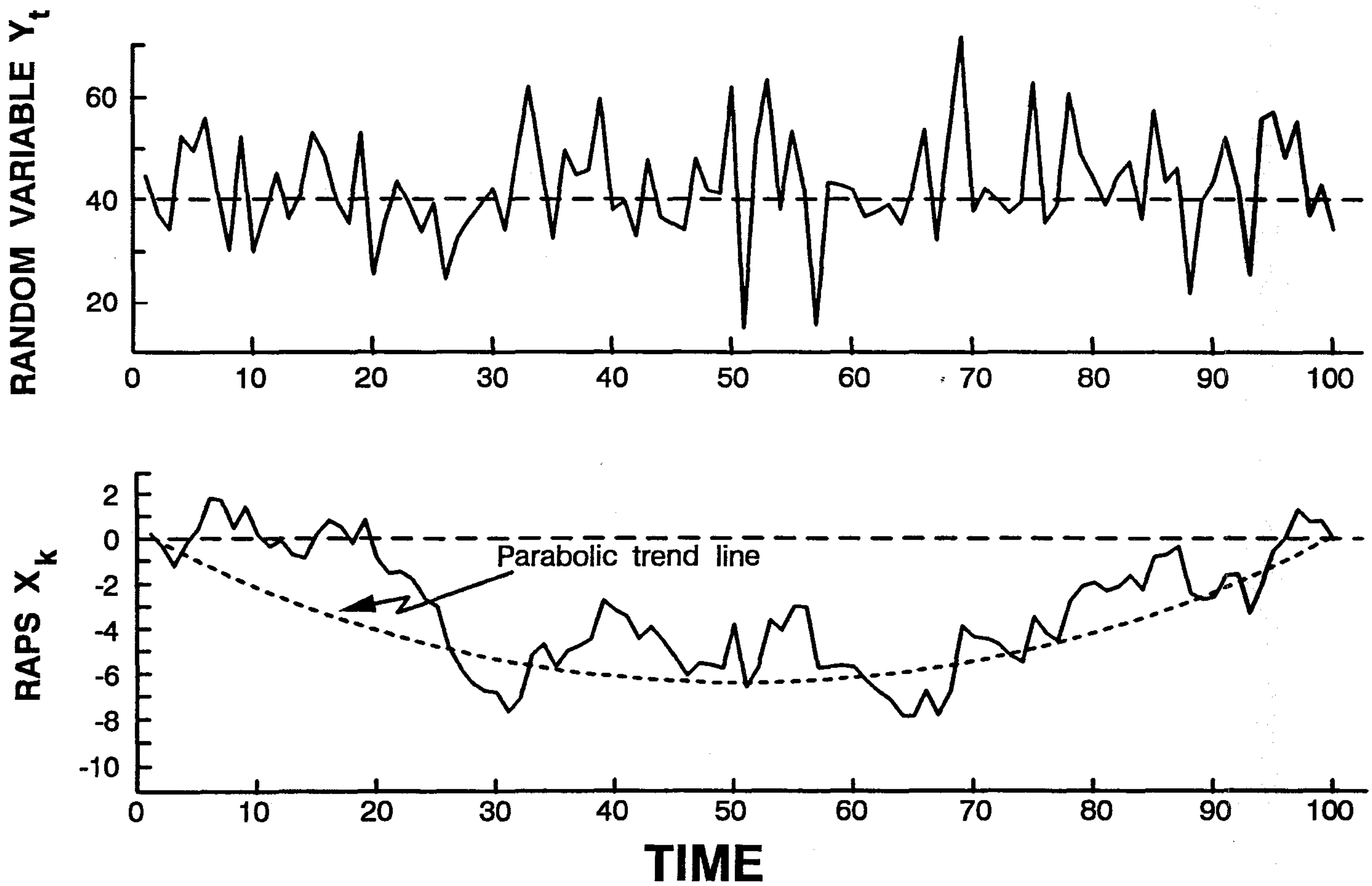

Figure 3. Time Series of n = 100 Values of Randomly Generated Y_t Values With a Uni-Directional Increasing Trend (top), and Corresponding Partial Sums X_k (bottom).

the expected value of the RAPS, as outlined in Cases 1, 2, and 3, are obtained. Alternatively, the pattern of fluctuations and trend in the RAPS may indicate the nature of the mean function, μ_t, of the time series. In practical applications, the nonstationarity of the time series Y_t is not known. Therefore, a plot of the RAPS simply reveals the possible existence of fluctuations and trends, and not their source or origin. The relative importance, significance, and implications of these fluctuations and trends must be interpreted from the plot within the context of the full record and confirmed by targeted statistical evaluations. In the following, the application of this generalized case is illustrated for two practical examples with unknown mean function μ_t.

APPLICATIONS

Annual Rainfall Series, Durant, Oklahoma

A 90-year annual rainfall record (1902-1991) is available for the Durant-USDA station (NWS Station # 2678), Bryan County, in south-central Oklahoma. The mean and standard deviation of the entire annual rainfall series are 1030 mm and 251 mm, respectively. The object of this application is to visualize long-term trends and the occurrence of wet and dry periods lasting up to 10 years. A plot of the annual rainfall values versus time (Figure 4a) shows considerable year-to-year variability. No particular fluctuations or trends are apparent, even though five out of the last six years are at least one standard deviation above the mean.

A plot of the RAPS of the annual rainfall values (Figure 4b) shows several smaller fluctuations and two major trends. The first trend is defined by a general decrease starting in 1902 and ending in 1966 with a maximum departure of about 10 standard deviations. The second trend is defined by an increase from 1967 and 1991. A double-mass-curve of the Durant record and the south-central divisional annual rainfall record (Climatological Data Annual Summary for Oklahoma; NCDC) shows a relative change in the two records beginning in the late 1950s. This is also the time when the station was relocated from downtown to the campus of Southeastern State College (November 1955). It is likely that the shift was the result of the relocation of the station and the Durant record was adjusted accordingly using the double-mass-curve technique described by Chow (1964) and Linsley (1949).

A plot of the RAPS of the adjusted record for Durant is shown in Figure 4c. The previously discussed trend is significantly reduced, leaving primarily a series of fluctuations of differing duration and magnitude. Most of the rising and falling limbs are related to flood and drought occurrences that have previously been reported in the literature (Table 1). This application shows that the visualization not only indicated the existence of a shift in the record (Figure 4b), but occurrences of floods and drought are clearly defined by fluctuations in the RAPS. Neither the shift nor the periods of floods and droughts can be easily recognized in the standard plot of the time series of the annual rainfall amounts.

Annual Two-Day Maximum Rainfall Series, New Orleans, Louisiana

An analysis of the temporal fluctuations of heavy rainfall magnitudes in New Orleans, Louisiana, from 1871 through 1991 was conducted by Keim and Muller (1992). The same data set is analyzed here with the objective of plotting the corresponding RAPS and comparing the resulting trends and fluctuations with their findings. A plot of the annual two-day maximum rainfall series is reproduced at the top of Figure 5. The long-term mean of the series is 145 mm with a standard deviation of 66 mm. From this plot, Keim and Muller (1992) determined four periods with unique characteristics:

> "The period from 1871 through 1926 appears to differ from the rest of the series with extreme events between 200 and 250 mm occurring every six or seven years on the average. The period from 1927 through 1948 experienced storms of unprecedented magnitude, including two of the greater magnitudes of the series. Extreme annual maxima then dropped in magnitude and never exceeded 197 mm from 1949 through 1977. Since 1978, the number of extreme rain events has been uncharacteristically large . . . ."

Tests by Keim and Muller (1992) showed that there were no trends or long-term persistence in rainfall magnitude. In addition, storms of the most recent 14-year period were comparable to those recorded during the 1927-1948 period.

The RAPS of the annual two-day maximum rainfall series is shown at the bottom of Figure 5. The four periods defined by Keim and Muller (1992) are much easier to identify by the RAPS than by a standard time series plot. The first period is defined by a steady

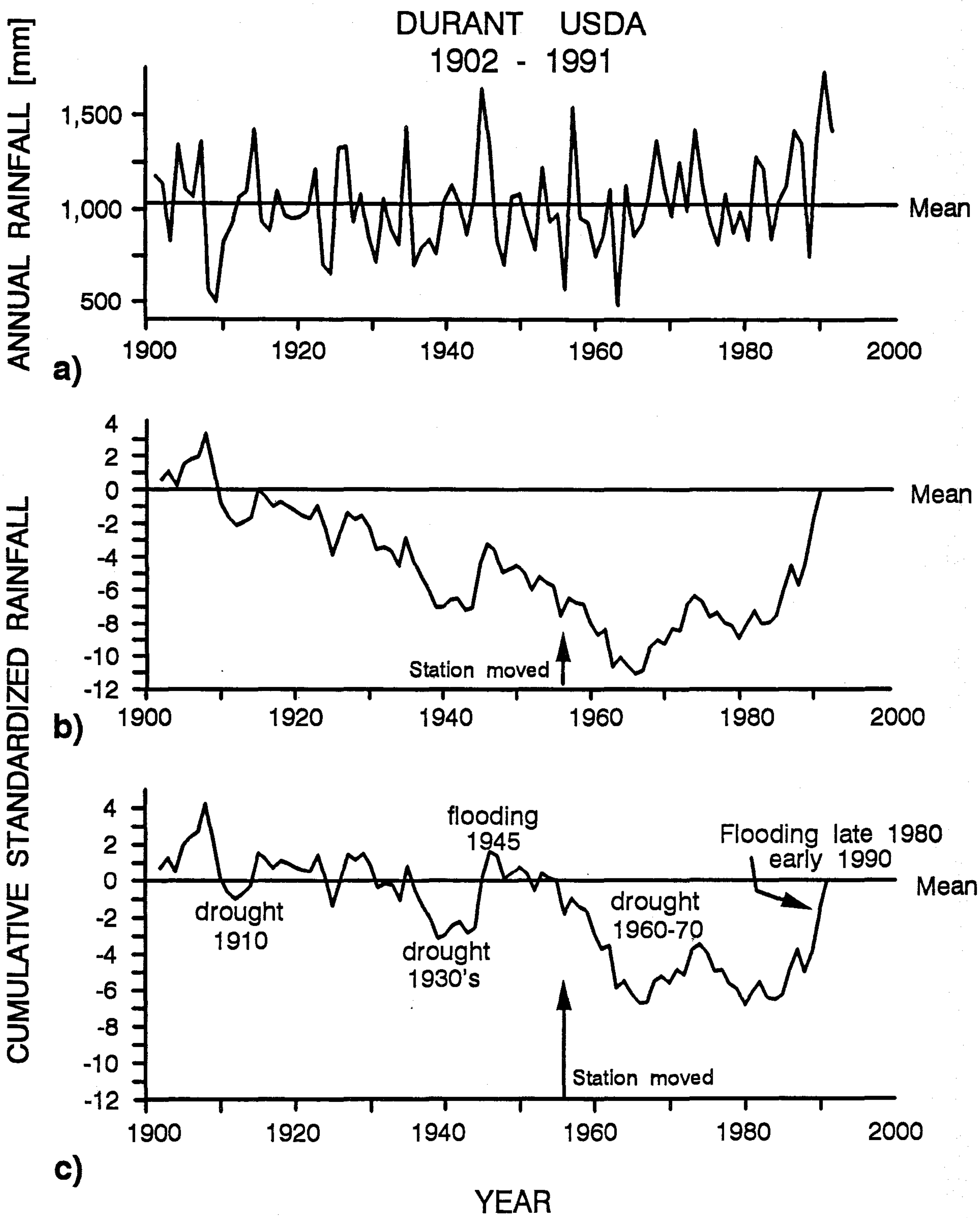

Figure 4. Annual Rainfall Values for the Durant Station (1902-1991): a) Original Record; b) Rescaled Adjusted Partial Sums; and c) Rescaled Adjusted Partial Sums for Corrected Record. The arrow on the time scale in b) and c) indicate the time at which the Durant Station was moved.

TABLE 1. Reported Floods and Droughts in Oklahoma from 1901 to 1987.

Date	Flood/Drought	Affected Area	Source
1910-1913	Drought	Great Plains	Ludlum (1971)
1923	Floods	North Central Oklahoma	USGS (1991)
1930-1940	Drought	South-Central Plains	Ludlum (1971)
1943-1950	Floods	East Central Oklahoma	USGS (1991)
1952-1957	Drought	Northwest Texas, Southern Plains, and Eastward	Ludlum (1971)
1961-1972	Drought	Oklahoma State-Wide	USGS (1991)
1975-1982	Drought	Oklahoma State-Wide	USGS (1991)
1981-1987	Floods	South-Central and Southwestern Oklahoma	USGS (1991)

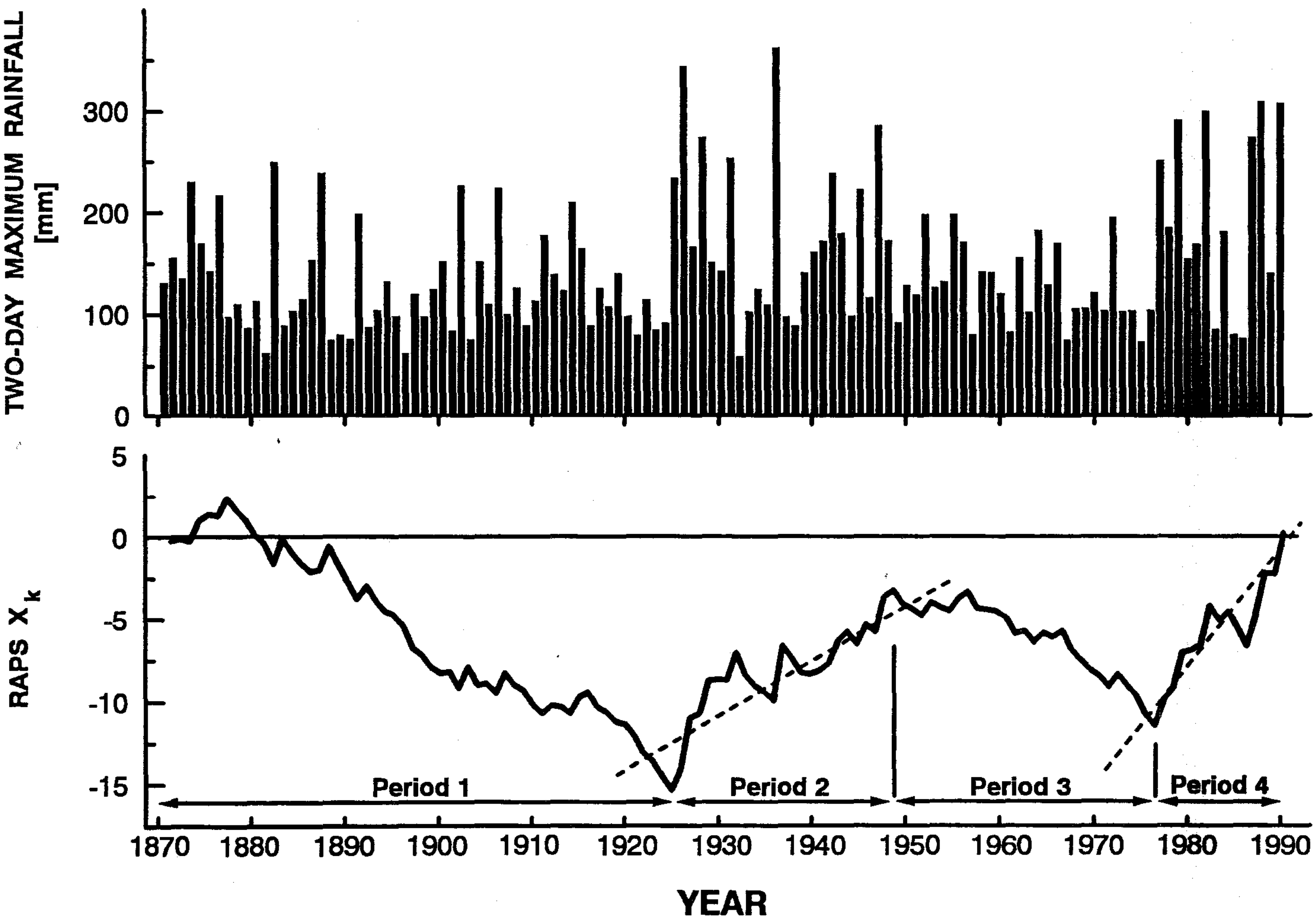

Figure 5. Two-Day Maximum Rainfall Series at New Orleans (top), and Corresponding r.a. Partial Sums (Bottom).

decreasing trend, followed by an increasing trend for the second period, a renewed decline for the third, and an increase for the fourth. The departures produced by the trends are up to 15 standard deviations and the timing of changes in trends are nearly identical to those defined by Keim and Muller (1992). Based on the RAPS, the periods could be defined slightly differently as 1871-1925, 1925-1949, and 1950-1977, respectively. Finally, the RAPS do not show the existence of a long-term trend towards higher rainfall values with time, which supports the findings of Keim and Muller (1992).

The graphical interpretation of the period 1926-1949 with that of 1978-1991 shows a few interesting features. First, the departures produced by both periods are of about the same magnitude (12 standard deviations). This indicates similar cumulative rainfall volumes (using the annual two-day maximums only) for each period. Second, the duration of the periods is quite different. Given the similar magnitudes of the departures, this leads to a different rate of change in the trends (as indicated by the dashed lines in Figure 5). Even though the two highest two-day rainfall occurred in the period 1926-1949, the period 1978-1991 has the steeper slope, meaning that the effect of the two highest two-day rainfall is overshadowed by the larger number of successive high rainfall events over a shorter time. In this example, the frequency of rainfall events above 250 mm for the period of 1978-1991 is about twice that of the period of 1926-1949. This application shows that the visualized fluctuations shown in Figure 5 not only agree with the findings of Keim and Muller (1992), but also reveal differences in rainfall frequencies between the 1926-1949 and 1978-1991 periods.

DISCUSSIONS AND CONCLUSIONS

The preceding applications demonstrate the usefulness of the RAPS approach to visualize trends and fluctuations in climatic records. The rescaling and summation of the RAPS approach highlights small yet systematic changes over time that are often hidden in a standard time series plot by the comparatively large magnitude and variability of the data itself. The approach is best suited for trends that are continuous throughout the record, or that change at some time in the middle of the record. Changes in trends located at the edges of a record cannot easily be distinguished from normally occurring fluctuations. Also, fluctuations are best reproduced if they span three or more time increments. Data clustering and persistent behavior are very well recognized. Shorter fluctuations are generally canceled as a result of the summation process. Therefore, isolated extreme values, even though large by themselves, are not necessarily prevalent in the visualization by use of RAPS. They can, however, easily be recognized in the standard time series plot. Finally, the visualization can highlight sequences of fluctuations, their distribution in time, and possible periodicities in the record.

The approach is not without limitations. First, and most important, the visualization itself is not a substitute for statistical evaluations. It allows a preliminary or exploratory inspection of the data and provides qualitative information to focus and to assist planning and subsequent analysis of the record. It can also be used after the analysis to check for additional features that were not detected by the analysis, or to visually display the findings of the analysis. Second, as mentioned earlier, the use of RAPS may suggest the presence of large periodicities that do not really exist because higher order partial sums converge to the sinusoidal limit (Klemes *et al.*, 1988). The impact of this effect is minimized because: (a) only the first-order partial sums are used; (b) climatic variables are generally not the result of cumulative processes in the same sense as water level in a reservoir or glacier volumes; and (c) as stated earlier, visualized features should be statistically tested for significance using the original time series.

Given a careful application of the RAPS, the visualization can be used in a complementary fashion with a standard time series plot and statistical test, and can lead to information on the number, magnitude, shape, frequency, and timing of fluctuations, and on the magnitude, start, and end of trends. Such information can guide and focus a subsequent statistical analysis of the record for trends, shifts, data clustering, periodicities, etc. Even though annual rainfall and two-day maximum rainfall were selected as application examples, the approach is believed to be applicable to other climatic variables and time increments.

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